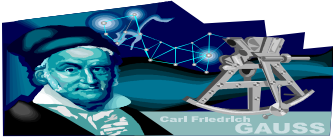


MA 110 COLLEGE ALGEBRA  
PRACTICE TEST THREE

Scens: 4.5, 4.6, 4.7, 5.1, 5.2, 5.3, 5.4, 5.5.



1. Find the vertical, horizontal, or oblique asymptotes of each rational function then graph the functions given. Verify your results using a graphing utility. State the domain of each function.

[4.5, 4.6]

a)  $G(x) := \frac{x}{x+2}$

b)  $G(x) := -3 \frac{1}{x^2 + 3x - 10}$

c)  $R(x) := \frac{x+1}{x(x+4)}$

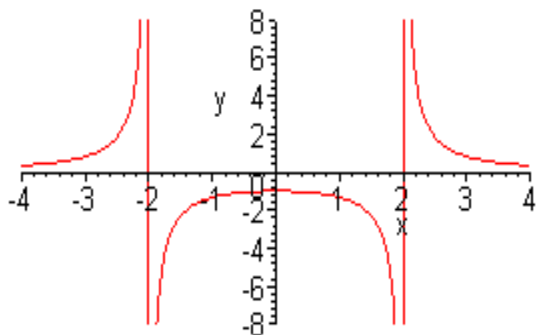
d)  $R(x) := \frac{x^3}{x^2 + 1}$

e)  $R(x) := \frac{x^2 - 1}{x - 1}$

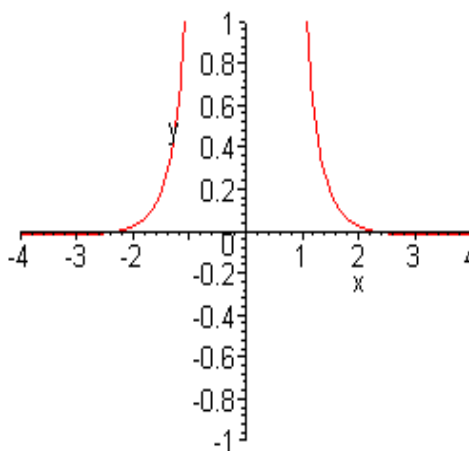
2-3. Use the graph below to find: [4.5]

- The domain and range of the function.
- Horizontal asymptotes, if any
- The oblique asymptotes, if any
- The intercepts, if any
- The vertical asymptotes, if any.

f) And the intervals over which the function is increasing or decreasing.



2.



3. 
$$G(x) := \frac{5 - x^2}{3x^4}$$

4. Sketch the graph by hand, and analyze the graph

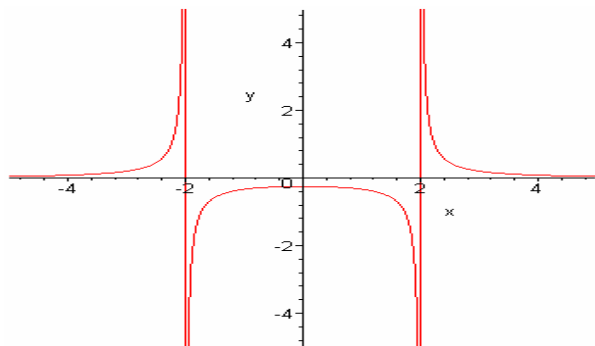
with the following: [4. 5, 4.6]

- Find the domain of the rational function.
- Locate the intercepts, if any, of the graph.
- Test for symmetry.
- Locate the horizontal, vertical, oblique asymptotes, and a “hole” on  $R(x)$ .
- Graph  $R(x)$ .

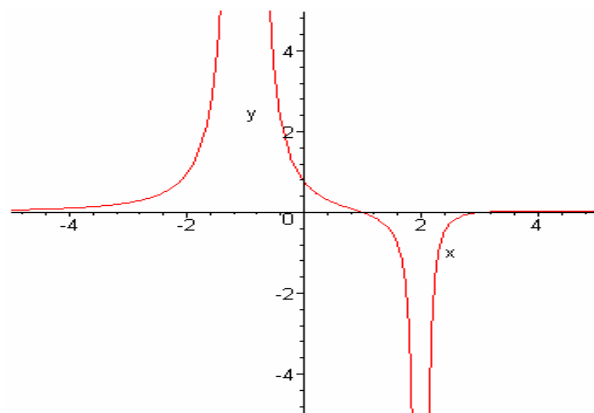
$$R(x) := \frac{x^2 + 3x - 10}{x^2 + 8x + 15}$$

5-6. Use the vertical asymptotes, x-intercepts, and their multiplicities to construct an equation that corresponds to each graph. Be sure the y-intercept on the graph matches the value given by your equation. Check work on a graphing calculator. [4, 5, 4, 6]

5.



6.



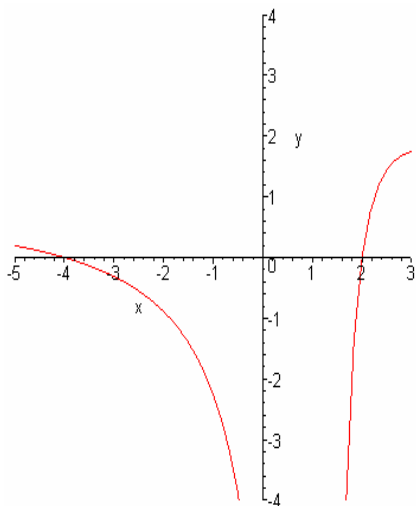
7. *Application:* In the following table,  $Y_1$ , is defined by a rational expression of the form  $\frac{x-p}{x-q}$ . Use the table to find the values of  $p$  and  $q$ . [4.5, 4.6]

x	y
-4	0.2
-3	0
-2	0.33
-1	-1
0	-3
1	error
2	5

8. Graph the following function. If there is a removable discontinuity, repair the break by redefining the function using an appropriate piecewise-defined function. [4.6]

$$f(x) := \frac{x^3 - 1}{x - 1}$$

9. Use the following graph to give the solution set of a)  $f(x) = 0$ , b)  $f(x) < 0$ , c)  $f(x) > 0$ . Use the set notation for part a) and interval notation for parts b) and c). [4.7]



10. Solve the inequality  $\frac{4x+5}{x+2} \geq 3$ , and graph the solution set. [4.7]

11. Solve each of the following. Use set notation for part (a). Then, use a graph to determine the solution sets of the associated inequalities in parts (b), (c), d) and e)

[4.7]

(a)  $\frac{3}{x-2} + \frac{1}{x+1} = \frac{1}{x^2 - x - 2}$

(b)  $\frac{3}{x-2} + \frac{1}{x+1} \leq \frac{1}{x^2 - x - 2}$

(c)  $\frac{3}{x-2} + \frac{1}{x+1} \geq \frac{1}{x^2 - x - 2}$

d)  $\sqrt{2x+4} + 2 > 4x$

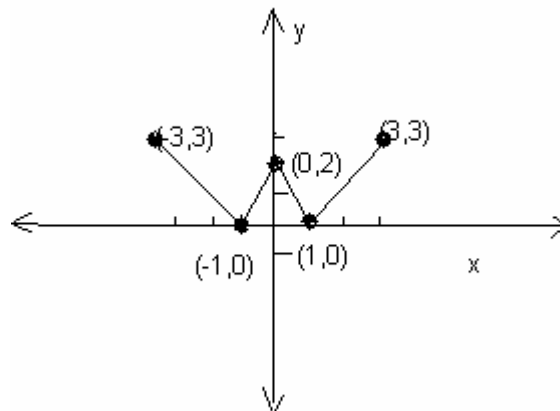
e)  $\sqrt[3]{4x^2 - 4x + 1} < 4x$

12. Compare the graph of each function with the graph of  $f(x) = \sqrt{x}$ . [5.1]

a)  $g(x) := -\sqrt{x}$

b)  $h(x) := \sqrt{-x}$

13- 16. *Applications*: Given the graph shown below, sketch by hand the graph of each function described, indicating how the five points labeled on the original have been



translated. [5.1]

13.  $f(x-1)$

14.  $f(x) + 1$

15.  $-3 f(x)$

16. Complete the second half of the graph of  $y = f(x)$  in the figure for each of the following conditions. [5.1]

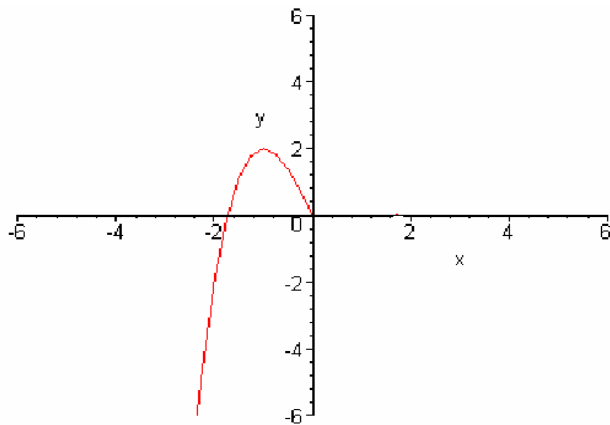
(a)  $f$  is odd.

(b)  $f$  is even.

(c)  $y = -f(x)$

(d)  $y = f(-x)$

(e)  $y = |-f(-x)|$



17. Graph the following function by translating the basic given function and clearly state what shifts are applied. [5.1, 5.3]

$$f(x) = b^x \qquad f(x) := \pi^{(-x+2)} - 3$$

18-21. Solve each exponential equation algebraically. Then check your answer by substituting into the original equation. [5.1, 5.4]

18.  $16^x = \frac{1}{8}$

19.  $27^{(x-2)} = 9$

20.  $e^{(x-1)} = 3^x$

21.  $\left(\frac{1}{2}\right)^x = \frac{1}{4}$

22. **Alcohol and Driving** The concentration of alcohol in a person's blood is measurable. Recent medical research suggests that the risk  $R$  (given as a percent) of having an accident while driving a car can be modeled by the equation

$$R = 6e^{kx}$$

where  $x$  is the variable concentration of alcohol in the blood and  $k$  is a constant. [5. 1, 5.2]

a) Suppose that a concentration of alcohol in the blood of 0.04 results in a 10% risk ( $R=10$ ) of an accident. Find the constant  $k$  in the equation.

b) Using this value of  $k$ , what is the risk if the concentration is 0.17?

c) Using the same value of  $k$ , what concentration of alcohol corresponds to a risk of 100%?

The loudness  $L(x)$ , measured in decibels, of a sound of intensity  $x$ , measured in watts per square meter, is defined as  $L(x) = 10 \log \frac{x}{I_0}$ , where  $I_0 = 10^{-12}$  watt per square meter is the least intense sound that a human ear can detect. Determine the loudness, in decibels, of the following sounds. [5. 1, 5.2]

23. **Loudness of Sound** Normal conversation: intensity of  $x = 10^{-7}$  watt per square meter.

24. Write each equation in exponential form. [5. 2]

a)  $\log_{10} \left( \frac{1}{100} \right) = -2$

b)  $\log_3 27 = x$

25. Write each equation in a logarithm form. [5. 2]

a)  $81^{1/4} = 3$

b)  $6^{-2} = 1/36$

26. Find exact value without using the calculator. [5. 2]

a)  $\log_{64}(-4)$

b)  $\log_{\frac{1}{8}} 16$

c)  $\log_6 36$

27. Solve each algebraically. Then check on a graphing calculator. [5. 2, 5.3, 5.4]

a)  $\log_3 (x-12) = 2$

b)  $\log_4 (x-2) + 2 \log_4 x = 1 + \log_4 2x$

c)  $25^{(2x)} = 5^{(x^2-12)}$

d)  $8 = 4^{(x^2)} 2^{(5x)}$

e)  $4^x - 2^x - 12 = 0$

f)  $e^x + 3e^{(-x)} = 4$

g)  $-0.345 = \ln\left(\frac{1}{x^3}\right)$

h)  $9^{5x-3} = 78462$

28. Determine the domain of the following function.

$$y = \log_4 \sqrt{7-4x}$$

29. Express in terms of sums and differences of logarithms. [5. 3]

a)  $\log \sqrt[3]{10x^2}$

$$\frac{\log \sqrt{x^2+4}}{x e^{(3x)}}$$

b)

30. Express as a single logarithm and simplify, if possible. [5. 3]

a)  $\frac{1}{2}\log_{10} x - 3\log_{10}(x+1)$

b)  $2\ln(x+2) - \ln(x)$

c)  $\frac{1}{3}[2\ln(x+3) + \ln(x) - \ln(x^2 - 1)]$

d)  $\log_a(3z+4) - \frac{2}{3}\log_a(z+2)$

31. Simplify. [5. 3]

a)  $\log_7 7^{(2x)}$

b)  $\ln e^{(x^2+2x)}$

c)  $\log_2 16^{(x-2)}$

32. Solve using any method. [5. 4]

a)  $\ln(x) + \ln(x^2 + 1) = 8$

b)  $1 + \log_4(-x) = 0$

c)  $\log(x+14) - \log(x) = \log(x+6)$

33-35. *Application:* Graph each function by hand and support your sketch with a calculator graph. Explain how the graph of each function can be obtained from the graph of  $y = 4^x$ . Give the domain, range, and equation of the asymptote. [5. 2]

33.  $h(x) := 3 - 4^{(-x)}$

34.  $g(x) := 4^{(x+2)}$

35.  $f(x) := \left(\frac{1}{4}\right)^x - 2$

36- 38. Find the amount that result from the following investment.

36. \$500 invested at 8% compounded quarterly after a period of  $2\frac{1}{2}$  years.

37. How long will it take \$4,000 to grow to \$9,000 if it is invested at 15% compounded monthly?

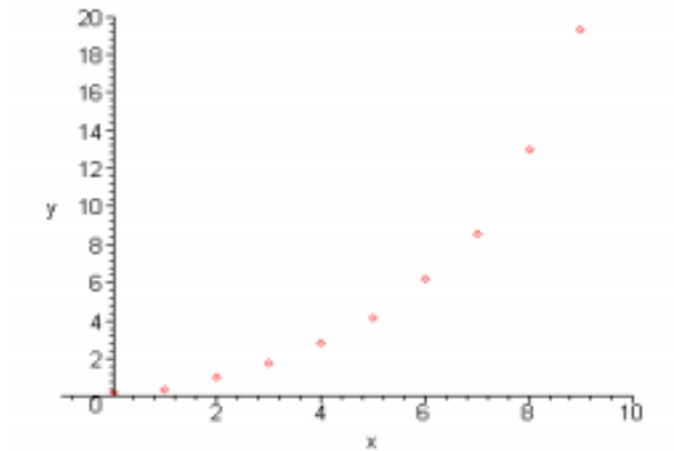
38. Which is the better investment and why 9% compounded monthly or 9.3% compounded annually?

39. Determine the time necessary for \$1000 to double if it is invested at interest rate 11% compounded [5. 2. 5. 3]

- a) daily
- b) continuously

40. (Modeling) Population Growth A population is increasing according to the equation

$$y = .304(1.627^x)$$



where  $y$  is in millions and  $t$  is in years. Match each question in parts a)-d) with one of the solution methods A-D. [5. 5]

- a) How long will it take for the population to triple?
- b) When will the population reach 3 million?
- c) How large will the population be in 3 years?
- d) How large will the population be in 4 months?

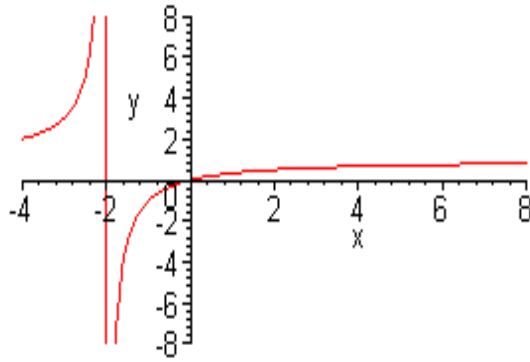
A. Evaluate  $0.304(1.627)^{1/3}$

B. Solve  $0.304(1.627)^x = 3(0.304)$  for  $x$ .

C. Evaluate  $0.304(1.627)^3$

D. Solve  $0.304(1.627)^x = 3$  for  $x$ .

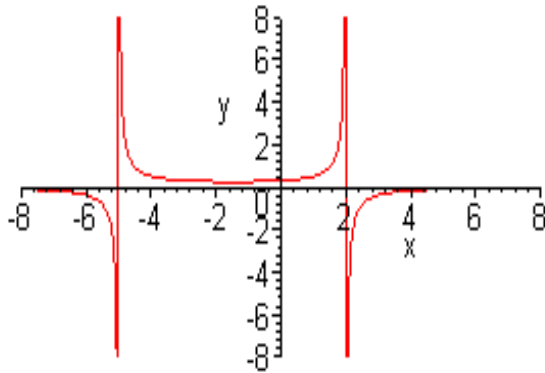
Answers:



1a)

V.A.  $x = -2$ ; H.A.  $y = 1$ ;

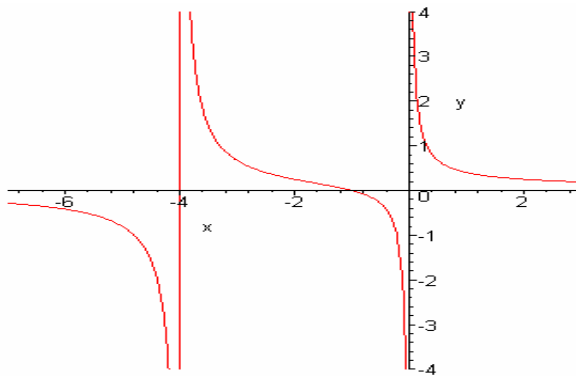
Domain:  $(-\infty, -2) \cup (-2, \infty)$



b)

V.A.  $x = -5, x = 2$ ; H.A.  $y = 0$ ;

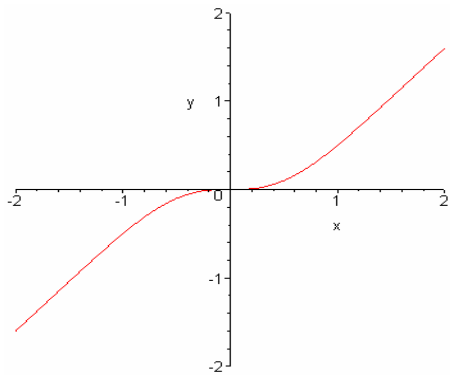
Domain:  $(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$



c)

V.A.:  $x = 0; x = -4$ ; H.A.  $y = 0$

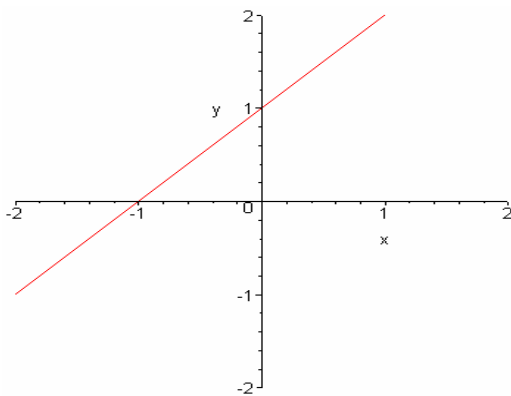
Domain:  $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$



d)

V.A. none; H.A. none; Oblique Asymptote:  $y = x$

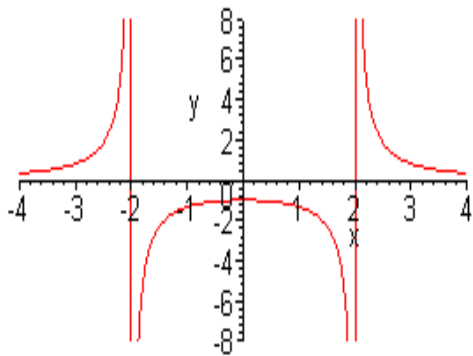
The domain of  $R(x)$  consists of all real solutions.



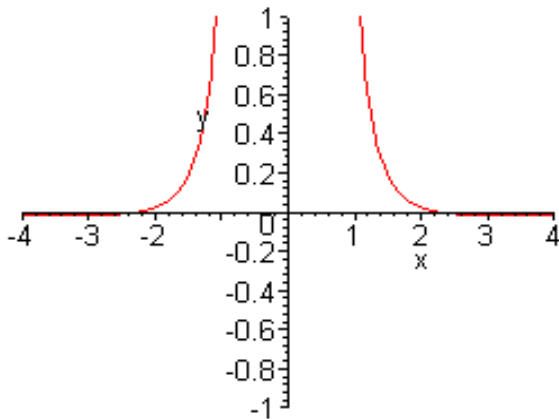
e)

V.A. None, Hole at one; H.A. None

The domain of  $R(x)$  consists of all real numbers except 1.

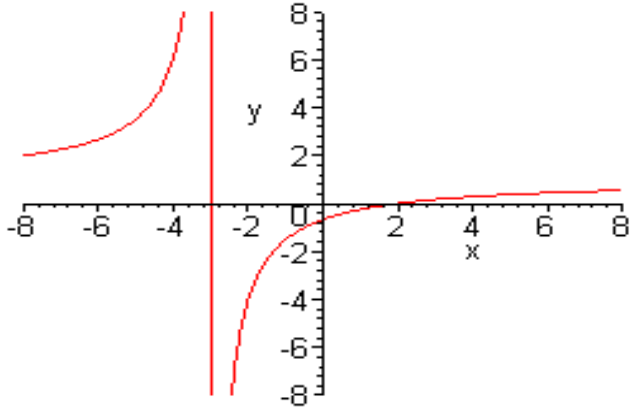


2. a) Domain:  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ ;  
 Range:  $(-\infty, -0.25) \cup (-0.25, \infty)$ ; b) H.A.  $y = 0$ ; c) O.A. none; d) y-intercept:  $-0.25$ ;  
 e) V.A.  $x = \pm 2$ ; f) Increasing:  $(-\infty, -2) \cup (-2, 0)$ ; Decreasing:  $(0, 2) \cup (2, \infty)$ ;



3. a) Domain:  $(-\infty, 0) \cup (0, \infty)$ ;  
 Range:  $(-0.016666, 0] \cup [0, \infty)$ ; b) H.A.  $y = 0$ ; c) O.A. : none; d) y-intercept: none; x-intercept:  $x \approx \pm 2.236$ ; e) V.A.  $x = 0$ ; f) Increasing:  $(-\infty, 0)$ ; Decreasing:  $(0, \infty)$ .

- 4a) Domain:  $\{x / x \neq -3, x \neq -5\}$ ;  $\beta$ ) x-intercept:  $x = 2$ ; y-intercept:  $y = -2/3$ ;  
 c) No symmetry. d) V.A.  $x = -3$ ; hole at  $(-5, 3.5)$ ; H.A.  $y = 1$ ; not intersected;

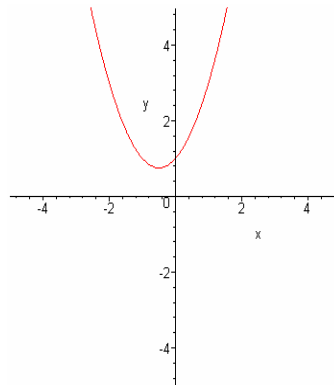


e)

5)  $G(x) := \frac{1}{x^2 - 4}$  (Answers may vary)

6)  $F(x) := \frac{(x-1)(x-3)}{(x+1)^2(x-2)^2}$  (Answers may vary)

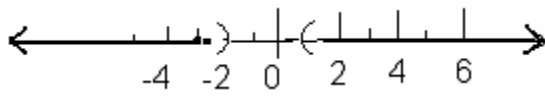
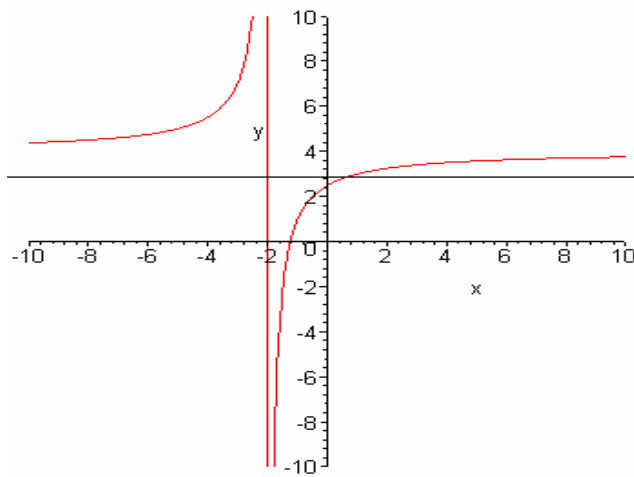
7) Ans:  $p = -3$ ;  $q = 1$ .



8) 
$$F(x) = \begin{cases} \frac{x^3 - 1}{x - 1} & x \neq 1 \\ 3 & x = 1 \end{cases}$$

9 a)  $\{-4, 2\}$ ; b)  $(-4, 2)$ ; c)  $(-\infty, -4) \cup (2, \infty)$ .

10. Ans:

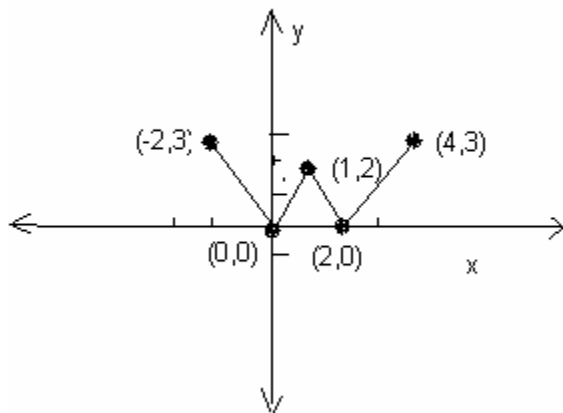


$(-\infty, -2) \cup [1, \infty)$ .

11(a)  $\{0\}$ ; (b)  $(-\infty, -1) \cup [0, 2)$ ; (c)  $(-1, 0] \cup (2, \infty)$ ; (d)  $[-2, \frac{9}{8}]$ ; (e)  $[0.184, \infty)$

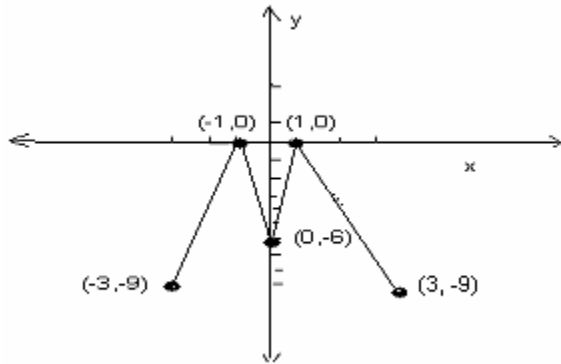
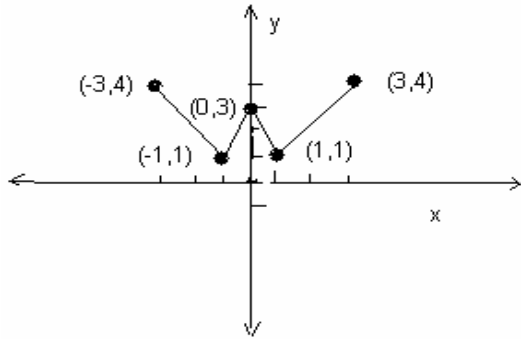
12. a)  $g(x) := -\sqrt{x}$  -----  $\rightarrow$  the graph is a reflection on the x-axis.

b)  $h(x) := \sqrt{-x}$  -----  $\rightarrow$  the graph is a reflection on the y-axis.

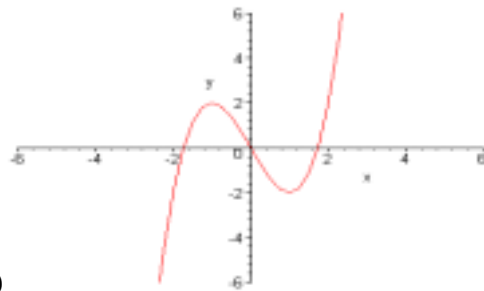


13.

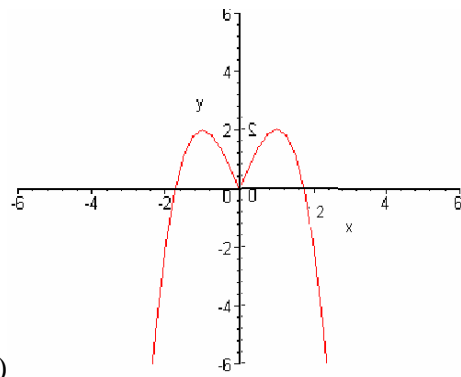
14.



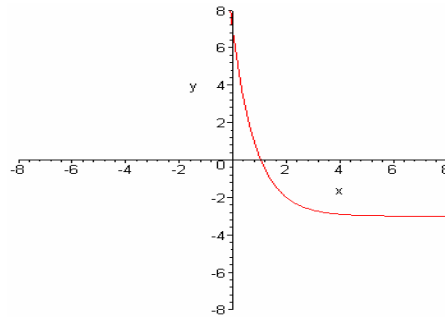
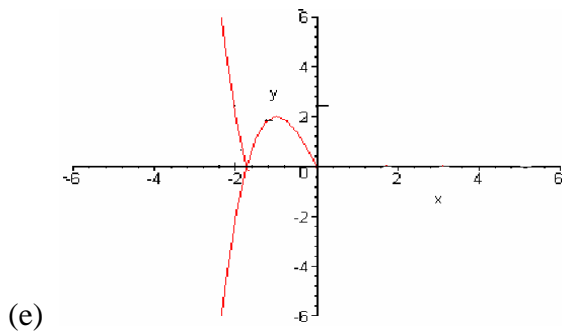
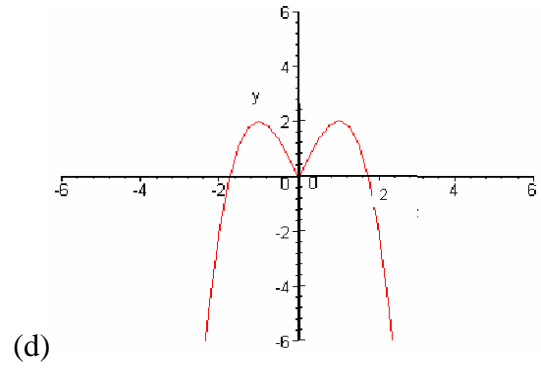
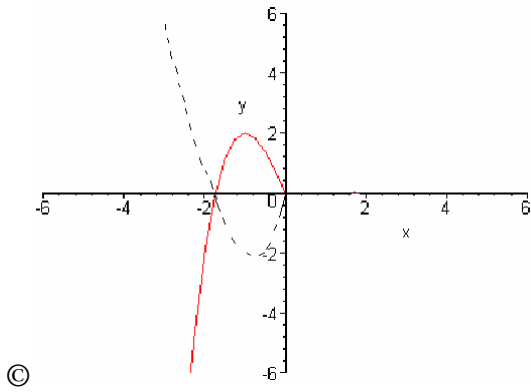
15.



16. (a)



(b)



17.  $f(x) := \pi^{(-x+2)} - 3$

reflect across the y-axis, horizontal shift right 2 units, vertical shift down 3 units.

18)  $x = -3/4$ ; 19)  $x = 8/3$ ; 20)  $x = \frac{1}{1 - \ln(3)} \approx -10.1407$ ; 21)  $x = 2$

22a)  $R = 6e^{kx} \rightarrow 10 = 6e^{(0.04k)}$ ;  $k = 12.77$

b)  $R = 6e^{kx} = 6e^{(12.77)(0.17)} = 52.6$  c)  $R = 6e^{kx} = 100 = 6e^{12.77x}$ ;  $x = 0.22$

23) 50 decibels    24 a))  $10^{-2} = 1/100$ ;    b)  $3^x = 27$

25a)  $\log_{81} 3 = \frac{1}{4}$ ;    b)  $\log_6 \frac{1}{36} = -2$     26 a) undefined;    b)  $-4/3$ ;    c) 2

27a)  $x = 21$ ; b)  $x = 4$ , reject  $x = -2$ ;    c)  $x = -2, x = 6$ ;    d)  $x = 1/2, x = -3$ ;    e)  $x = 2$ ;  
 $x = \emptyset$  f)  $x = 0$ ;  $x \approx 1.099$ ;

g)  $x = \ln(3) - 0.345 = \ln\left(\frac{1}{x^3}\right) \quad x = \sqrt[3]{e^{-0.345}} \quad x \approx 1.122$

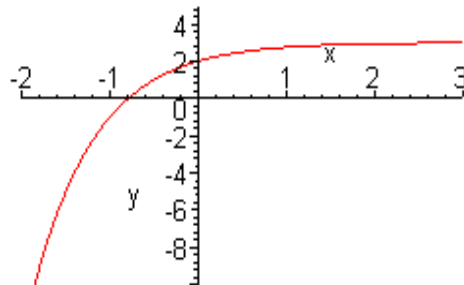
h)  $x = \frac{\ln 78462}{5 \ln 9} + \frac{3}{5} \quad x \approx 1.6259$

28)  $x \in (-\infty, \frac{7}{4})$     29a)  $\frac{1}{3}(1 + 2 \log(x))$     b)  $\frac{1}{2} \ln(x^2 + 4) - \ln(x) - 3x$

30a)  $\log_{10} \frac{\sqrt{x}}{(x+1)^3}$     b)  $\ln \frac{(x+2)^2}{x}$     c)  $1 \ln^3 \sqrt{\frac{x(x+3)^2}{x^2-1}}$     d)  $\log_a \frac{(3z+4)}{(z+2)^{2/3}}$

31a)  $2x$     b)  $x^2 + 2x$     c)  $4x - 8$

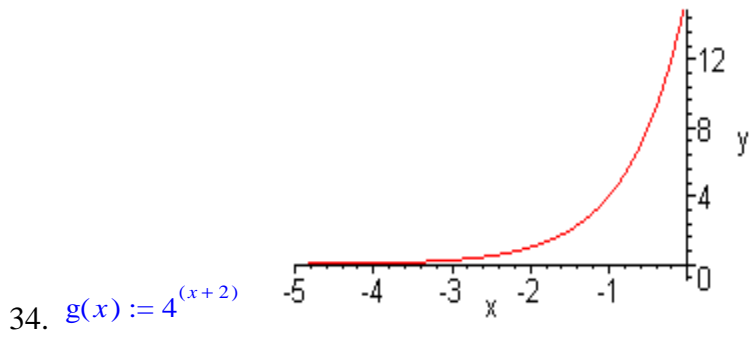
32a)  $x = 14.369$ ;    b)  $x = -1/4$     c)  $x=2$ ;  $-7$  is extraneous



33.  $h(x) := 3 - 4^{(-x)}$

Reflect the graph  $y = 4^x$  across the y-axis; reflect across the x-axis and shift up three units.

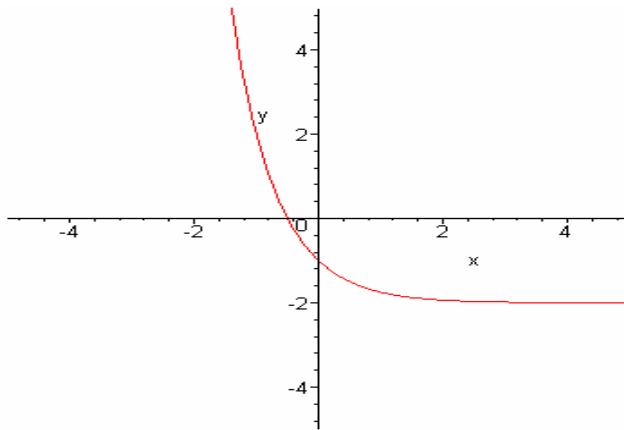
Domain: All real numbers; Range:  $(-\infty, 3)$ ; H.A.  $y = 3$



Shift the graph  $y = 4^x$  to the left 2 units.

Domain: All real numbers; Range:  $(0, \infty)$ ; H.A.  $y = 0$

35.  $f(x) := \left(\frac{1}{4}\right)^x - 2$



Reflect the graph  $y = 4^x$  across the  $y$ -axis; shift down 2 units.

Domain: All real numbers; Range:  $(-2, \infty)$ ; H.A.  $y = -2$

36) \$609.50; 37) 5.44 yrs

38) 9% compounded monthly, since its effective rate is 9.38%; the effective rate of 9.3% compounded annually is 9.3%.

39a) 6.302 yr; b) 6.301yr

40 a)-B; b) - D; c) - C; d) - A

