

Table entry for z is the area to the left of z .

Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

For values of z less than -3.49 , use 0.000 to approximate the area.

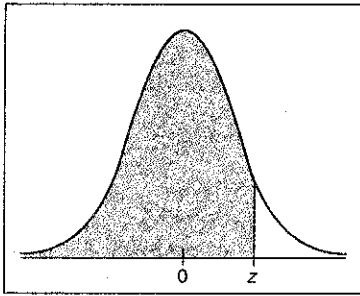


Table entry for z is the area to the left of z .

Areas of a Standard Normal Distribution *continued*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For z values greater than 3.49, use 1.000 to approximate the area.

Areas of a Standard Normal Distribution *continued*

(b) Confidence Interval Critical Values z_c	
Level of Confidence c	Critical Value z_c
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

Areas of a Standard Normal Distribution *continued*

(c) Hypothesis Testing, Critical Values z_0		
Level of Significance	$\alpha = 0.05$	$\alpha = 0.01$
Critical value z_0 for a left-tailed test	-1.645	-2.33
Critical value z_0 for a right-tailed test	1.645	2.33
Critical values $\pm z_0$ for a two-tailed test	± 1.96	± 2.58

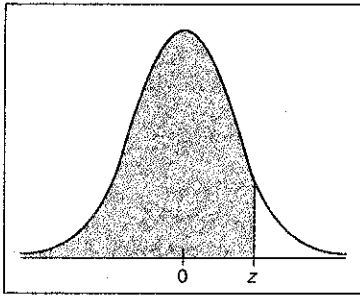


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2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
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FREQUENTLY USED FORMULAS

n = sample size N = population size f = frequency

$$\text{Class Width} = \frac{\text{high} - \text{low}}{\text{number classes}} \text{ (increase to next integer)}$$

$$\text{Class Midpoint} = \frac{\text{upper limit} + \text{lower limit}}{2}$$

$$\text{Lower boundary} = \text{lower boundary of previous class} + \text{class width}$$

$$\text{Sample mean } \bar{x} = \frac{\sum x}{n}$$

$$\text{Population mean } \mu = \frac{\sum x}{N}$$

$$\text{Weighted average} = \frac{\sum xw}{\sum w}$$

$$\text{Range} = \text{largest data value} - \text{smallest data value}$$

$$\text{Sample standard deviation } s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

$$\text{Computation formula } s = \sqrt{\frac{SS_x}{n - 1}} \text{ where}$$

$$SS_x = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$\text{Population standard deviation } \sigma = \sqrt{\frac{\sum(x - \mu)^2}{N}}$$

$$\text{Sample variance } s^2$$

$$\text{Population variance } \sigma^2$$

$$\text{Sample Coefficient of Variation } CV = \frac{s}{\bar{x}} \cdot 100$$

$$\text{Sample mean for grouped data } \bar{x} = \frac{\sum xf}{n}$$

$$\text{Sample standard deviation for grouped data}$$

$$s = \sqrt{\frac{\sum(x - \bar{x})^2 f}{n - 1}} = \sqrt{\frac{\sum x^2 f - (\sum xf)^2 / n}{n - 1}}$$

Regression and Correlation

In all these formulas

$$SS_x = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$SS_y = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

Least squares line $y = a + bx$ where $b = \frac{SS_{xy}}{SS_x}$ and

$$a = \bar{y} - b\bar{x}$$

Pearson product-moment correlation coefficient

$$r = \frac{SS_{xy}}{\sqrt{SS_x SS_y}}$$

Coefficient of determination = r^2

Probability of the complement of event A

$$P(\text{not } A) = 1 - P(A)$$

Multiplication rule for independent events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

General multiplication rules

$$P(A \text{ and } B) = P(A) \cdot P(B, \text{ given } A)$$

$$P(A \text{ and } B) = P(B) \cdot P(A, \text{ given } B)$$

Addition rule for mutually exclusive events

$$P(A \text{ or } B) = P(A) + P(B)$$

General addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Permutation rule $P_{n,r} = \frac{n!}{(n-r)!}$

Combination rule $C_{n,r} = \frac{n!}{r!(n-r)!}$

Sample Test Statistics for Tests of Hypotheses

for paired differences $d \quad t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ with $d.f. = n - 1$

difference of means large sample

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

difference of means small sample with $\sigma_1 \approx \sigma_2$;
 $d.f. = n_1 + n_2 - 2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{where } s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

difference of proportions

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}} \text{ where } \hat{p} = \frac{r_1 + r_2}{n_1 + n_2}; \hat{q} = 1 - \hat{p};$$

$$\hat{p}_1 = r_1/n_1; \hat{p}_2 = r_2/n_2$$

Confidence Intervals

for difference of means (when $n_1 \geq 30$ and $n_2 \geq 30$)

$$(\bar{x}_1 - \bar{x}_2) - z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

for difference of means ($n_1 < 30$ and/or $n_2 < 30$ and $\sigma_1 \approx \sigma_2$)
 $d.f. = n_1 + n_2 - 2$

$$(\bar{x}_1 - \bar{x}_2) - t_c s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_c s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\text{where } s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

for difference of proportions

$$\text{where } \hat{p}_1 = r_1/n_1; \hat{p}_2 = r_2/n_2; \hat{q}_1 = 1 - \hat{p}_1; \hat{q}_2 = 1 - \hat{p}_2$$

$$(\hat{p}_1 - \hat{p}_2) - z_c \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_c \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$\chi^2 = \sum \frac{(O - E)^2}{E} \text{ where } E = \frac{(\text{row total})(\text{column total})}{\text{sample size}}$$

Tests of Independence $d.f. = (R - 1)(C - 1)$

Goodness of fit $d.f. = (\text{number of entries}) - 1$

Sample test statistic for $H_0: \sigma^2 = k$; $d.f. = n - 1$

$$\chi^2 = \frac{(n - 1)s^2}{\sigma^2}$$

Linear Regression

$$\text{Standard error of estimate } S_e = \sqrt{\frac{SS_y - bSS_{xy}}{n - 2}}$$

$$\text{where } b = \frac{SS_{xy}}{SS_x}$$

Confidence interval for y

$y_p - E < y < y_p + E$ where y_p is the predicted y value for x and

$$E = t_c S_e \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{SS_x}} \text{ with } d.f. = n - 2$$

Mean of a discrete probability distribution $\mu = \sum(x)P(x)$

Standard deviation of a discrete probability distribution

$$\sigma = \sqrt{\sum(x - \mu)^2 P(x)}$$

For Binomial Distributions

r = number of successes; p = probability of success;

$$q = 1 - p$$

Binomial probability distribution $P(r) = C_{n,r} p^r q^{n-r}$

Mean $\mu = np$

Standard deviation $\sigma = \sqrt{npq}$

Raw score $x = z\sigma + \mu$

Standard score $z = \frac{x - \mu}{\sigma}$

Mean of \bar{x} distribution $\mu_{\bar{x}} = \mu$

Standard deviation of \bar{x} distribution $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Standard score for \bar{x} $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

Confidence Interval

for μ ($n \geq 30$)

$$\bar{x} - z_c \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_c \frac{\sigma}{\sqrt{n}}$$

for μ ($n < 30$)

$$d.f. = n - 1$$

$$\bar{x} - t_c \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_c \frac{s}{\sqrt{n}}$$

for p ($np > 5$ and $nq > 5$)

$$\hat{p} - z_c \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} < p < \hat{p} + z_c \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

where $\hat{p} = r/n$

Sample Size n for Estimating

means $n = \left(\frac{z_c \sigma}{E}\right)^2$

proportions

$n = p(1 - p) \left(\frac{z_c}{E}\right)^2$ with preliminary estimate for p

$n = \frac{1}{4} \left(\frac{z_c}{E}\right)^2$ without preliminary estimate for p

Sample Test Statistics for Tests of Hypotheses

for μ ($n \geq 30$) $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

for μ ($n < 30$); $d.f. = n - 1$ $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

for p $z = \frac{\hat{p} - p}{\sqrt{pq/n}}$ where $q = 1 - p$