

## FREQUENTLY USED FORMULAS

$n$  = sample size     $N$  = population size     $f$  = frequency

$$\text{Class Width} = \frac{\text{high} - \text{low}}{\text{number classes}} \text{ (increase to next integer)}$$

$$\text{Class Midpoint} = \frac{\text{upper limit} + \text{lower limit}}{2}$$

$$\text{Lower boundary} = \text{lower boundary of previous class} + \text{class width}$$

$$\text{Sample mean } \bar{x} = \frac{\sum x}{n}$$

$$\text{Population mean } \mu = \frac{\sum x}{N}$$

$$\text{Weighted average} = \frac{\sum xw}{\sum w}$$

$$\text{Range} = \text{largest data value} - \text{smallest data value}$$

$$\text{Sample standard deviation } s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$\text{Computation formula } s = \sqrt{\frac{SS_x}{n - 1}} \text{ where}$$

$$SS_x = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$\text{Population standard deviation } \sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

$$\text{Sample variance } s^2$$

$$\text{Population variance } \sigma^2$$

$$\text{Sample Coefficient of Variation } CV = \frac{s}{\bar{x}} \cdot 100$$

$$\text{Sample mean for grouped data } \bar{x} = \frac{\sum xf}{n}$$

$$\text{Sample standard deviation for grouped data}$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2 f}{n - 1}} = \sqrt{\frac{\sum x^2 f - (\sum xf)^2 / n}{n - 1}}$$

## Regression and Correlation

In all these formulas

$$SS_x = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$SS_y = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

Least squares line  $y = a + bx$  where  $b = \frac{SS_{xy}}{SS_x}$  and

$$a = \bar{y} - b\bar{x}$$

Pearson product-moment correlation coefficient

$$r = \frac{SS_{xy}}{\sqrt{SS_x SS_y}}$$

Coefficient of determination =  $r^2$

Probability of the complement of event A

$$P(\text{not } A) = 1 - P(A)$$

Multiplication rule for independent events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

General multiplication rules

$$P(A \text{ and } B) = P(A) \cdot P(B, \text{ given } A)$$

$$P(A \text{ and } B) = P(B) \cdot P(A, \text{ given } B)$$

Addition rule for mutually exclusive events

$$P(A \text{ or } B) = P(A) + P(B)$$

General addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Permutation rule  $P_{n,r} = \frac{n!}{(n-r)!}$

Combination rule  $C_{n,r} = \frac{n!}{r!(n-r)!}$

Mean of a discrete probability distribution  $\mu = \sum(x)P(x)$

Standard deviation of a discrete probability distribution

$$\sigma = \sqrt{\sum(x - \mu)^2 P(x)}$$

For Binomial Distributions

$r$  = number of successes;  $p$  = probability of success;

$$q = 1 - p$$

Binomial probability distribution  $P(r) = C_{n,r} p^r q^{n-r}$

Mean  $\mu = np$

Standard deviation  $\sigma = \sqrt{npq}$

Raw score  $x = z\sigma + \mu$

Standard score  $z = \frac{x - \mu}{\sigma}$

Mean of  $\bar{x}$  distribution  $\mu_{\bar{x}} = \mu$

Standard deviation of  $\bar{x}$  distribution  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Standard score for  $\bar{x}$   $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

Confidence Interval

for  $\mu$  ( $n \geq 30$ )

$$\bar{x} - z_c \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_c \frac{\sigma}{\sqrt{n}}$$

for  $\mu$  ( $n < 30$ )

$$d.f. = n - 1$$

$$\bar{x} - t_c \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_c \frac{s}{\sqrt{n}}$$

for  $p$  ( $np > 5$  and  $nq > 5$ )

$$\hat{p} - z_c \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} < p < \hat{p} + z_c \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

where  $\hat{p} = r/n$

Sample Size  $n$  for Estimating

means  $n = \left(\frac{z_c \sigma}{E}\right)^2$

proportions

$n = p(1 - p) \left(\frac{z_c}{E}\right)^2$  with preliminary estimate for  $p$

$n = \frac{1}{4} \left(\frac{z_c}{E}\right)^2$  without preliminary estimate for  $p$

Sample Test Statistics for Tests of Hypotheses

for  $\mu$  ( $n \geq 30$ )  $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

for  $\mu$  ( $n < 30$ );  $d.f. = n - 1$   $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

for  $p$   $z = \frac{\hat{p} - p}{\sqrt{pq/n}}$  where  $q = 1 - p$

Sample Test Statistics for Tests of Hypotheses

for paired differences  $d \quad t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$  with  $d.f. = n - 1$

difference of means large sample

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

difference of means small sample with  $\sigma_1 \approx \sigma_2$ ;  
 $d.f. = n_1 + n_2 - 2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{where } s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

difference of proportions

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}} \text{ where } \hat{p} = \frac{r_1 + r_2}{n_1 + n_2}; \hat{q} = 1 - \hat{p};$$

$$\hat{p}_1 = r_1/n_1; \hat{p}_2 = r_2/n_2$$

Confidence Intervals

for difference of means (when  $n_1 \geq 30$  and  $n_2 \geq 30$ )

$$(\bar{x}_1 - \bar{x}_2) - z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

for difference of means ( $n_1 < 30$  and/or  $n_2 < 30$  and  $\sigma_1 \approx \sigma_2$ )  
 $d.f. = n_1 + n_2 - 2$

$$(\bar{x}_1 - \bar{x}_2) - t_c s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_c s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\text{where } s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

for difference of proportions

where  $\hat{p}_1 = r_1/n_1; \hat{p}_2 = r_2/n_2; \hat{q}_1 = 1 - \hat{p}_1; \hat{q}_2 = 1 - \hat{p}_2$

$$(\hat{p}_1 - \hat{p}_2) - z_c \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_c \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$\chi^2 = \sum \frac{(O - E)^2}{E} \text{ where } E = \frac{(\text{row total})(\text{column total})}{\text{sample size}}$$

Tests of Independence  $d.f. = (R - 1)(C - 1)$

Goodness of fit  $d.f. = (\text{number of entries}) - 1$

Sample test statistic for  $H_0: \sigma^2 = k; d.f. = n - 1$

$$\chi^2 = \frac{(n - 1)s^2}{\sigma^2}$$

Linear Regression

$$\text{Standard error of estimate } S_e = \sqrt{\frac{SS_y - bSS_{xy}}{n - 2}}$$

$$\text{where } b = \frac{SS_{xy}}{SS_x}$$

Confidence interval for y

$y_p - E < y < y_p + E$  where  $y_p$  is the predicted y value for x and

$$E = t_c S_e \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{SS_x}} \text{ with } d.f. = n - 2$$